Mohr Circle Review + Examples

A. Given a state of stress where:

$\sigma_1 = 40\ MPa = \sigma_{zz}$

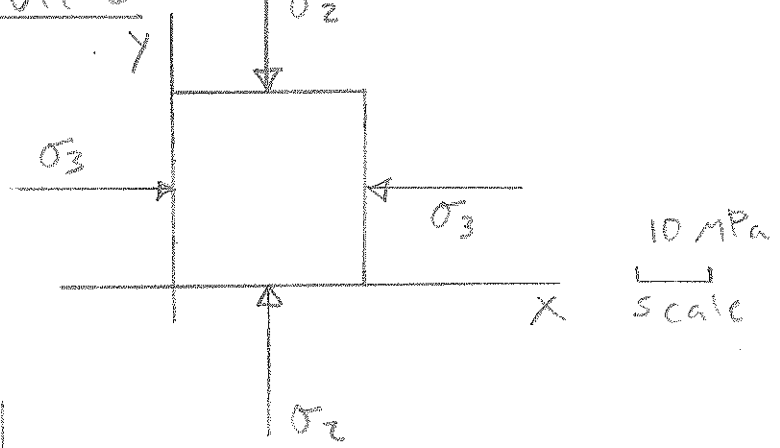
$\sigma_2 = 20\ MPa = \sigma_{yy}$

$\sigma_3 = 20\ MPa = \sigma_{xx}$

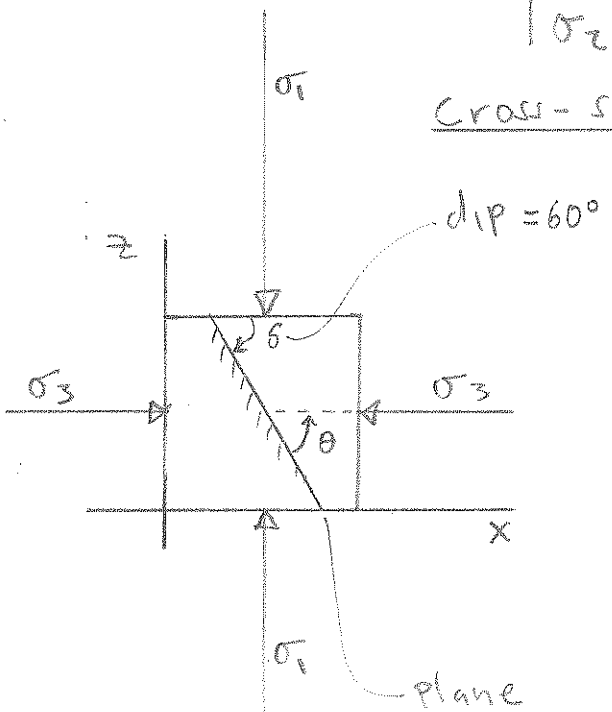
What are the normal and shear stresses acting on a plane dipping 60° towards x-positive and striking ⊥ to x.

1. Sketch the geometry of the problem

Map View



Cross-sectional View



1.2 Measure θ from the plane to the σ_3 axis. Positive angles are measured clounter-clockwise (like in physics).

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A. Given a state of stress where

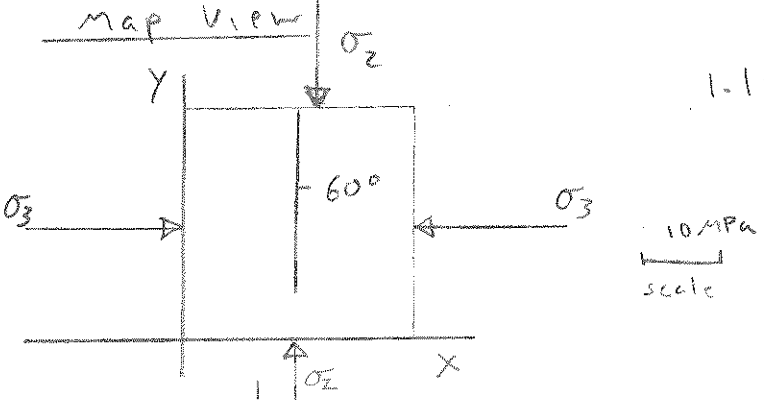
$\sigma_1 = 40$ MPa $= \sigma_{zz}$

$\sigma_2 = 20$ MPa $= \sigma_{yy}$ $\quad\quad$ $\sigma_3 = 20$ MPa $= \sigma_{xx}$

What are the normal and shear stresses resolved on a plane dipping 60° towards x-positive and strikes ⊥ to x.

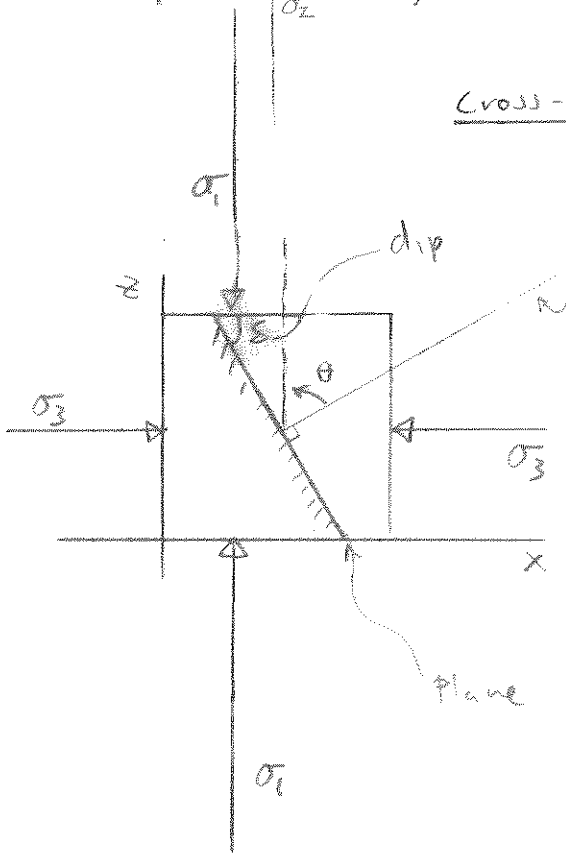
1. Sketch the geometry of the problem.

Map View



1.1 Scale the vectors correctly

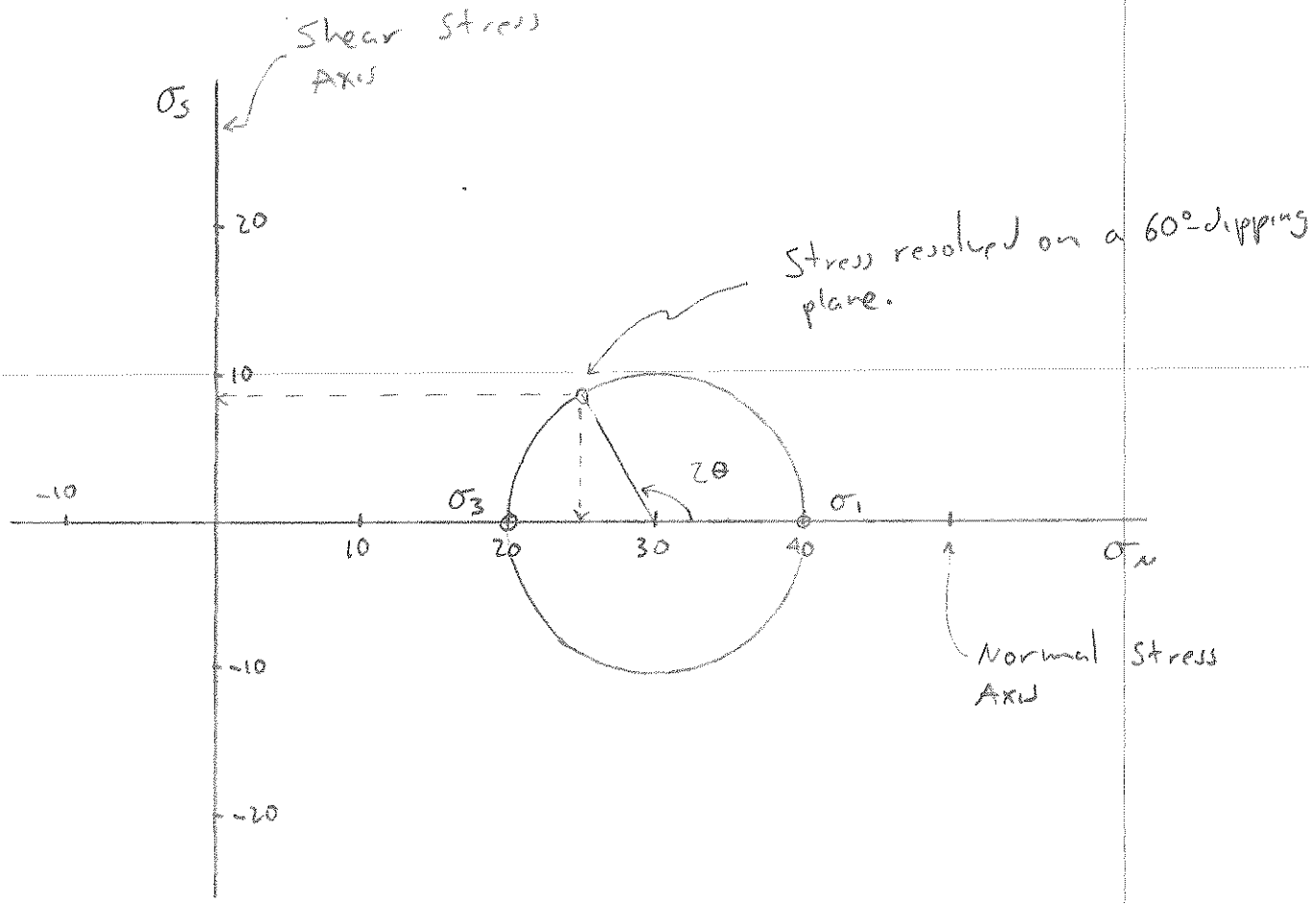
Cross-sectional View



$\delta = 60° = \theta$

1.2 Measure θ from the normal to the plane to the σ_1 direction. Positive angles are counter-clockwise (like in physics).

2. Plot Mohr Circle for this state of stress:



2.1. Plot σ_1 and σ_3 along the normal stress axis

2.2 find the center of the circle midway between σ_1 and σ_3. $= \frac{\sigma_1 + \sigma_3}{2}$ = (mean stress)

2.3 Draw the circle using a compass.

2.4 Plot 2θ from σ_N axis. Counter clockwise = positive angle

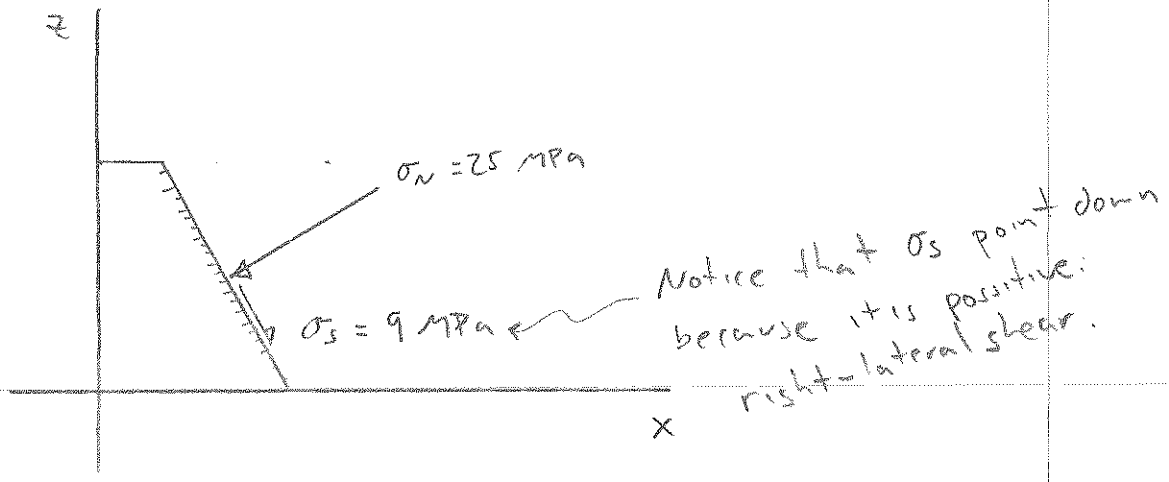
In this case $2\theta = 120°$

2.5 Read the normal and shear stress on the axis.

$\sigma_N = \sim 25$ MPa

$\sigma_s = \sim 9$ MPa

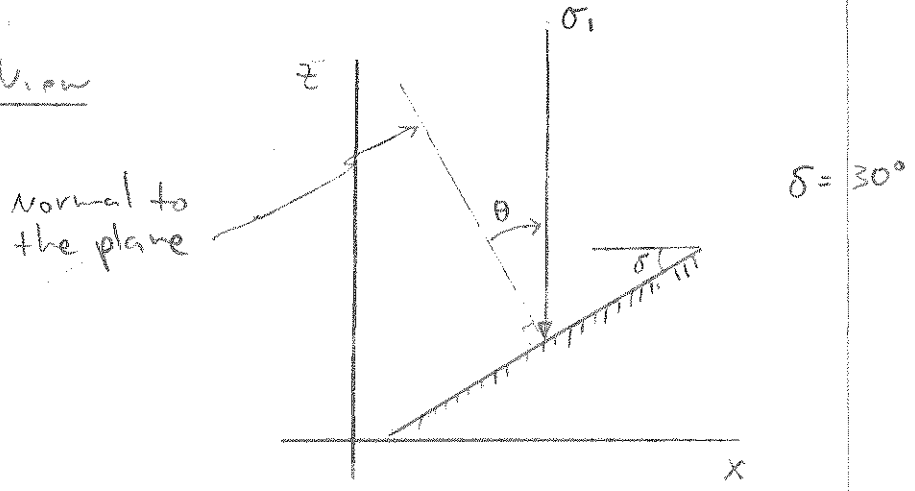
3. Plot the vectors on a sketch



B. What are the stresses resolved on a plane that dips 30° towards negative-x?

1. Sketch the problem:

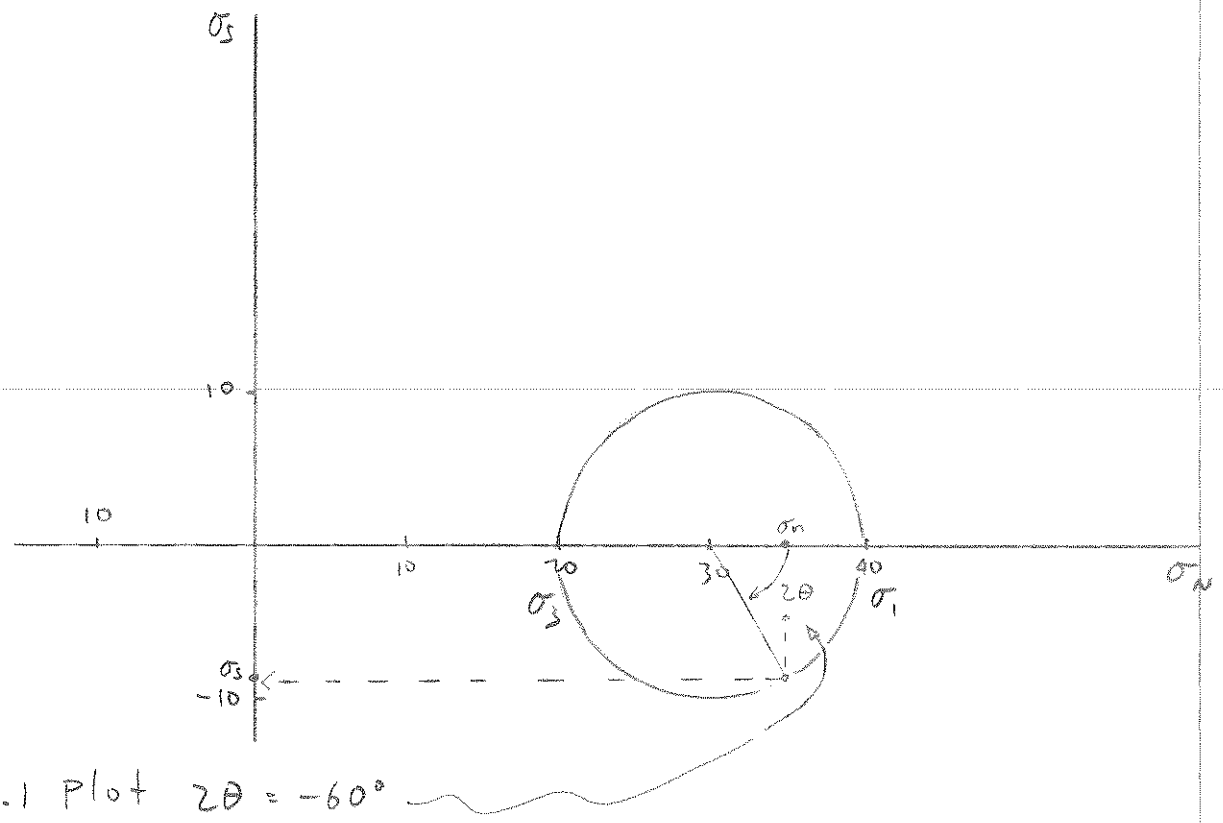
Cross-sectional View



1.1 Notice that θ is a negative angle

so $\theta = -30°$

2. Plot the Mohr circle (same as before)



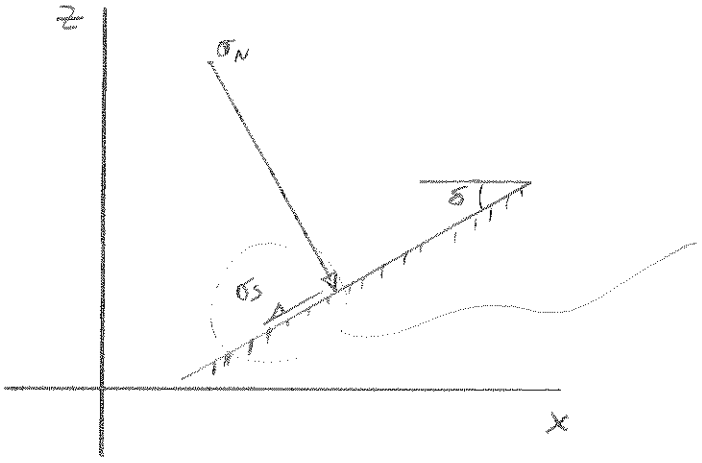
2.1 Plot $2\theta = -60°$

2.2 Read the normal and shear stresses

σ_n : ~35 MPa

σ_s : ~ −9 MPa

2.3 Plot the resolved vectors on your sketch.



σ_S indicates left-lateral shear because it is negative.

C. You can find the exact values of σ_N and σ_S by using the "fundamental stress equations":

$$\sigma_N = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta$$

$$\sigma_S = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta$$

for part A

$$\sigma_N = \frac{40\ MPa + 20\ MPa}{2} + \frac{40\ MPa - 20\ MPa}{2} \cos 120^\circ =$$

$$\boxed{\sigma_N = 25\ MPa}$$

$$\sigma_S = \frac{40\ MPa - 20\ MPa}{2} \sin 120^\circ$$

$$\boxed{\sigma_S = 8.66\ MPa}$$

for part B

$$\sigma_N = \frac{40\ MPa + 20\ MPa}{2} + \frac{40\ MPa - 20\ MPa}{2} \cos(-60^\circ)$$

$$\boxed{\sigma_N = 35\ MPa}$$

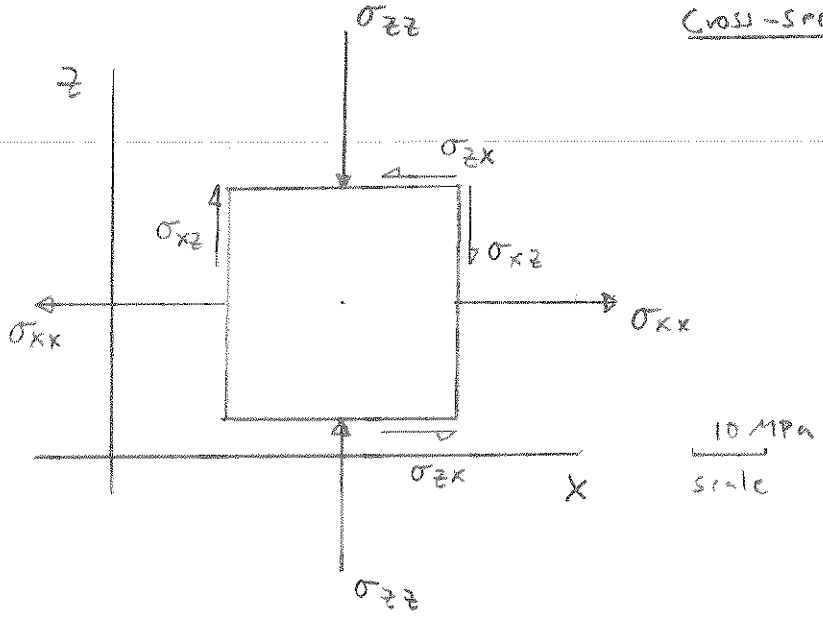
$$\sigma_S = \frac{40\ MPa - 20\ MPa}{2} \sin(-60)$$

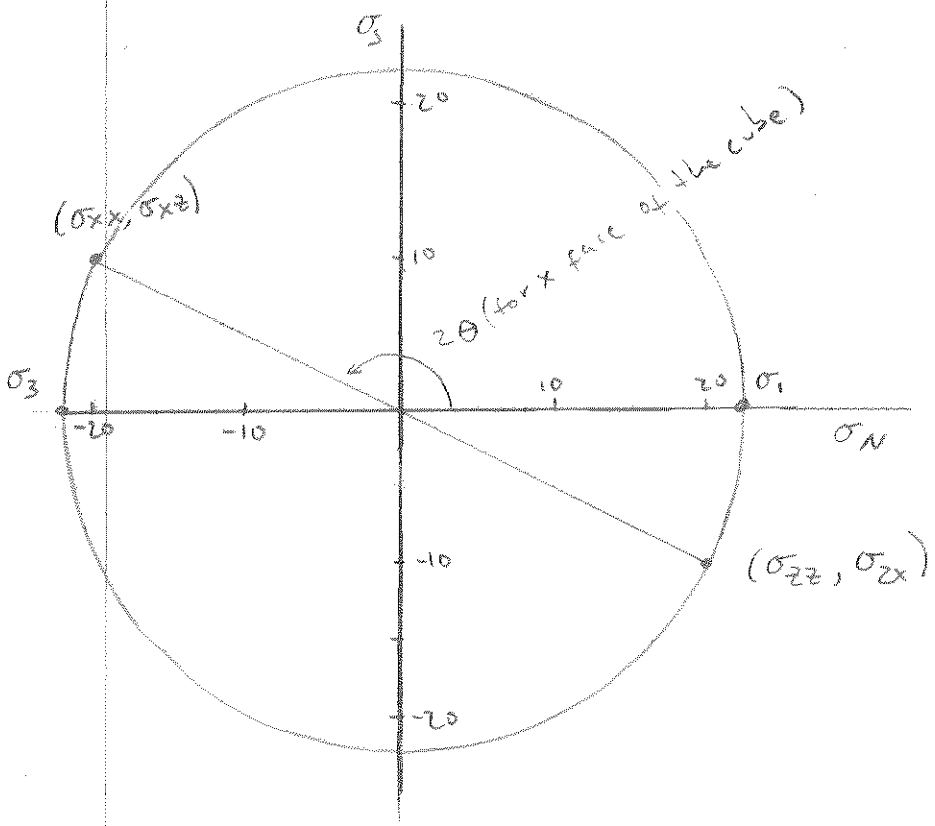
$$\boxed{\sigma_S = -8.66\ MPa}$$

D. Given $\sigma_{zz} = 20$ MPa, $\sigma_{xx} = -20$ MPa, $\sigma_{zx} = -10$ MPa, $\sigma_{xz} = 10$ MPa

find the magnitude and orientation of σ_1 and σ_3.

1. Sketch the stress field





2.1 Plot the stresses acting on two adjacent faces of the cube

2.2 Find the center of the Mohr circle by connecting the two pairs with a line.

2.3 Draw the Mohr circle

2.4 Read σ_1 and σ_3 along the σ_N axis.

$\sigma_1 = 22.5$ MPa

$\sigma_3 = -22.5$ MPa

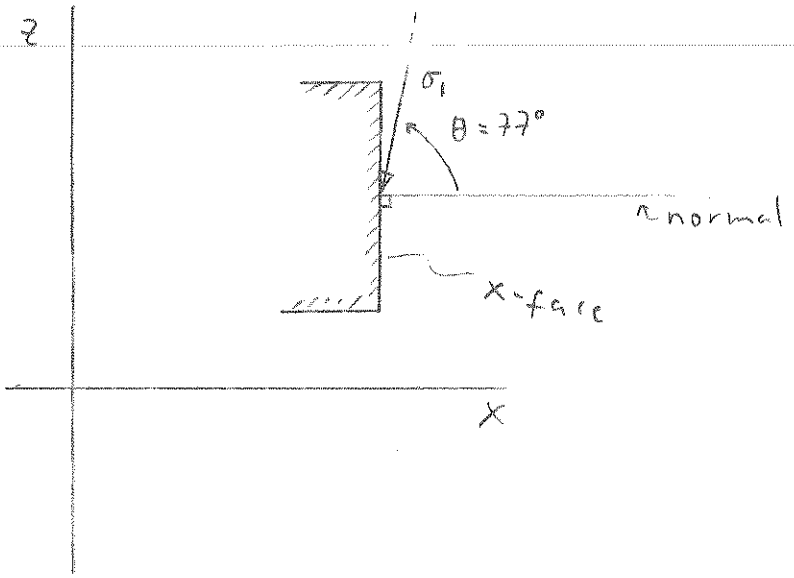
3. find the orientation of σ_1

3.1 Determine 2θ for x face of the cube

In this case $2\theta = 154°$

$\theta = 77°$

3.2 Sketch the cube again and plot θ.



So the principal stresses are oriented like this: